On the interplay between screening and confinement from interacting electromagnetic and torsion fields

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Features of screening and confinement are studied for the coupling of axial torsion fields with photons in the presence of an external electromagnetic field. To this end we compute the static quantum potential. Our discussion is carried out using the gauge-invariant but path-dependent variables formalism which is alternative to the Wilson loop approach. Our results show that, in the case of a constant electric field strength expectation value, the static potential remains Coulombic, while in the case of a constant magnetic field strength expectation value the potential energy is the sum of a Yukawa and a linear potential, leading to the confinement of static probe charges.

I. INTRODUCTION

The formulation and possible experimental consequences of extensions of the Standard Model (SM) such as torsion fields have been vastly investigated over the latest years [1, 2, 3, 4, 5, 6, 7, 8, 9]. As is well-known, this is because the SM does not include a quantum theory of gravitation. In fact, the necessity of a new scenario has been suggested to overcome difficulties theoretical in the quantum gravity research. In this respect we recall that string theories [10] provide a consistent framework to unify all fundamental interactions. We also point out that string theories are endowed with interesting features such as a metric, a scalar field (dilaton) and an antisymmetric tensor field of the third rank which is associated with torsion. It is worth recalling at this stage that, in addition to the string interest, torsion fields have been discussed under a number of different aspects. For instance, in connection to the observed anisotropy of the cosmological electromagnetic propagation [11, 12], the dark energy problem [13], and in higher dimensional theories [14, 15]. The ongoing activity of the Large Hadronic Collider (LHC) has also attracted interest in order to test the dynamical torsion parameters [16] and, related to this issue, the production of light gravitons [17, 18, 19, 20] at accelerators justifies the study of dynamical aspects of torsion.

On the other hand, in recent times the coupling of axial torsion fields with photons in the presence of an external background electromagnetic field and its physical consequences have been discussed [21]. Meanwhile, in a previous paper [22], the impact of axial torsion fields on physical observables in terms of the gauge-invariant but path-dependent variables formalism has been studied. Specifically, we have computed the static potential between test charges for a system consisting of a gauge field interacting with propagating axial torsion fields when there are nontrivial constant expectation values for the gauge field strength $F_{\mu\nu}$. According to this formalism, in the case of a constant electric field strength expectation value the static potential remains Coulombic. Interestingly enough, in the case of a constant magnetic field strength expectation value the potential energy is linear, that is, the confinement between static charges is obtained. In this case, the mass of torsion fields (spin-1 states) contribute to the string tension. The picture which emerges from this study is that the coupling of torsion fields with photons in the presence of a constant magnetic field strength expectation, behaves like small magnetic dipoles in an external magnetic field.

With these considerations in mind, the present work is a sequel to Ref. [22]. To do this, we will work out the static potential for a theory which includes both spin-1 and spin-0 states for the axial torsion field S_μ coupled to photons in the presence of an external background electromagnetic field, using the gauge-invariant but path-dependent variables formalism. Following this procedure we obtain that, in the case of a constant electric field strength expectation value the static potential remains Coulombic. While in the case of a constant magnetic field strength expectation value we obtain that the potential energy is the sum of a Yukawa and a linear potential, leading to the confinement of static charges. This clearly shows the role played by the spin-0 state of the torsion field S_μ in yielding the Yukawa potential. It is to be noted that the above static potential profile is analogous to that encountered in axionic electrodynamics [23]. Therefore, the above result reveals a new equivalence between effective Abelian theories. As well as, the gauge-invariant but path-dependent variables formalism offers an alternative view in which some features of effective Abelian theories

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become more transparent.

II. INTERACTION ENERGY

We shall now discuss the interaction energy between static point-like sources for the model under consideration. To this end, as in [22], we will compute the expectation value of the energy operator H in the physical state $|\Phi\rangle$ describing the sources, which we will denote by $\langle H \rangle_\Phi$. To carry out our study we consider the Lagrangian density [1, 21]:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{4}S_{\mu\nu}^2 + \frac{1}{2}m^2 S_\mu^2 - \frac{b}{2}(\partial_\mu S^\mu)^2 + \frac{g}{4}S^\lambda \partial_\lambda \left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right), \quad (1)$$

where $S_{\mu\nu} = \partial_\mu S_\nu - \partial_\nu S_\mu$, $\tilde{F}_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$, g is a coupling constant with dimension (-2) in mass units, and $b = \frac{m^2}{m_0^2}$. m and m_0, respectively, denote the masses of spin-1 and spin-0 states for the torsion field (S_μ).

According to the results of the paper of Ref. [8] on the constraints to be obeyed by quantum torsion, both the spin-1 and the spin-0 excitations of S_μ must be much more massive than the fundamental particles of the Standard Model. Also, if we assume that the spin-0 mode is much heavier than the spin-1 component of S_μ, the presence of the $(\partial_\mu S^\mu)^2$-term in (1) becomes harmless, in that the ghost mode that would run into troubles with unitarity is suppressed [21]. Then, considering the situation for which

$$m_0^2 \gg m^2 \gg m_{SM}^2, \quad (2)$$

where m_{SM}^2 stands for the masses of the fundamental particles of the Standard Model, we are allowed to integrate over the S_μ-field and consider an effective model that describes physics at scales $\ll m^2$.

Next, by integrating out the S_μ field in expression (1), one gets an effective Lagrangian density:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{g^2}{8}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)\frac{\Delta}{(b\Delta + m^2)}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right), \quad (3)$$

where $\triangle \equiv \partial^\mu \partial_\mu$. Now, after splitting $F_{\mu\nu}$ in the sum of a classical background, $\langle F_{\mu\nu}\rangle$, and a small fluctuation, $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, the corresponding Lagrangian density is given by

$$\mathcal{L} = -\frac{1}{4}f_{\mu\nu}^2 + \frac{g^2}{8}\left(v^{\mu\nu}f_{\mu\nu}\right)\frac{\Delta}{(b\Delta + m^2)}\left(v^{\lambda\gamma}f_{\lambda\gamma}\right). \quad (4)$$

Here, we have simplified our notation by setting $\varepsilon^{\mu\nu\alpha\beta}\langle F_{\mu\nu}\rangle \equiv v^{\alpha\beta}$. This effective theory thus provides us with a suitable starting point to study the interaction energy. It is straightforward to check that in the limit $b = 0$, expression (4) reduces to our previous effective Lagrangian [22].

A. Magnetic case

We now proceed to obtain the interaction energy in the $v^{0i} \neq 0$ and $v^{ij} = 0$ case (referred to as the magnetic one in what follows). In such a case, the Lagrangian density (4) reads as below:

$$\mathcal{L} = -\frac{1}{4}f_{\mu\nu}^2 + \frac{g^2}{8}\left(v^{0i}f_{0i}\right)\frac{\Delta}{(b\Delta + m^2)}\left(v^{0k}f_{0k}\right), \quad (5)$$

where $\mu, \nu = 0, 1, 2, 3$ and $i, k = 1, 2, 3$. We now restrict our attention to the Hamiltonian framework of this theory. The canonical momenta read $\Pi^\mu = f^{\mu 0} + \frac{g^2}{4}v^{0\mu}\frac{\Delta}{(b\Delta + m^2)}v^{0k}f_{0k}$. This yields the usual primary constraint $\Pi^0 = 0$, while the momenta are $\Pi_i = D_{ij}E_j$. Here $E_i \equiv f_{i0}$ and $D_{ij} = \delta_{ij} + \frac{g^2}{4}v_{i0}\frac{\Delta}{(b\Delta + m^2)}v_{j0}$. Since $\mathbf{D}$ is nonsingular, there exists its inverse, $\mathbf{D}^{-1}$. With this, the corresponding electric field can be written as

$$E_i = \frac{1}{\det D}\left\{\delta_{ij}\det D - \frac{g^2}{4}v_{i0}\frac{\Delta}{(b\Delta + m^2)}v_{j0}\right\}\Pi_j. \quad (6)$$

Therefore, the canonical Hamiltonian takes the form

$$H_C = \int d^3x \left\{-a_0\partial_i\Pi^i + \frac{1}{2}\mathbf{B}^2\right\} - \int d^3x \frac{1}{2}\Pi_i \left[1 - \frac{g^2\mathbf{v}^2}{4}\frac{\Delta}{(\xi^2\Delta + m^2)}\right]\Pi^i, \quad (7)$$

with $\xi^2 \equiv b + \frac{\mathbf{v}^2 g^2}{4} = b + \mathbf{v}^2 g^2 \mathcal{B}^2$. Here, $\mathbf{B}$ and $\mathcal{B}$ stand, respectively, for the fluctuating magnetic field and the classical background magnetic field around which the a^μ-field fluctuates. $\mathbf{B}$ is associated to the quantum a^μ-field: $B^i = -\frac{1}{2}\varepsilon_{ijk}f^{jk}$, whereas $\mathcal{B}_i$, according to our definition for the background $\langle F_{\mu\nu}\rangle$ in terms of $v_{\mu\nu}$ is given by $\mathcal{B}_i = \frac{1}{2}v_{0i}$. Temporal conservation of the primary constraint Π_0 leads to the secondary constraint $\Gamma_1(x) \equiv \partial_i\Pi^i = 0$. It is straightforward to check that there are no further constraints in the theory. Consequently, the extended Hamiltonian that generates translations in time then reads $H = H_C + \int d^3x \left(c_0(x)\Pi_0(x) + c_1(x)\Gamma_1(x)\right)$. Here $c_0(x)$ and $c_1(x)$ are arbitrary Lagrange multipliers. Moreover, it follows from this Hamiltonian that $\dot{a}_0(x) = [a_0(x), H] = c_0(x)$, which is an arbitrary function. Since $\Pi^0 = 0$ always, neither a^0 nor Π^0 are of interest in describing the system and may be discarded from the theory. As a result, the Hamiltonian becomes

$$H_C = \int d^3x \left\{c(x) + \frac{1}{2}\mathbf{B}^2\right\} - \int d^3x \frac{1}{2}\Pi_i \left[1 - \frac{g^2\mathbf{v}^2}{4}\frac{\Delta}{(\xi^2\Delta + m^2)}\right]\Pi^i, \quad (8)$$

where $c(x) = c_1(x) - a_0(x)$.

According to the usual procedure we introduce a supplementary condition on the vector potential such that

the full set of constraints becomes second class. A particularly convenient choice is found to be

$$\Gamma_2(x) \equiv \int_{C_{\xi x}} dz^\nu A_\nu(z) \equiv \int_0^1 d\lambda x^i A_i(\lambda x) = 0, \qquad (9)$$

where λ $(0 \leq \lambda \leq 1)$ is the parameter describing the spacelike straight path $x^i = \xi^i + \lambda(x-\xi)^i$, and ξ is a fixed point (reference point). There is no essential loss of generality if we restrict our considerations to $\xi^i = 0$. The choice (9) leads to the Poincaré gauge [24, 25]. As a consequence, the only nontrivial Dirac bracket for the canonical variables is given by

$$\begin{aligned}
\left\{A_i(x), \Pi^j(y)\right\}^* &= \delta_i^j \delta^{(3)}(x-y) \\
&\quad - \partial_i^x \int_0^1 d\lambda x^j \delta^{(3)}(\lambda x - y). \quad (10)
\end{aligned}$$

We are now equipped to compute the interaction energy for the model under consideration. As mentioned before, in order to accomplish this purpose we will calculate the expectation value of the energy operator H in the physical state $|\Phi\rangle$. Let us also mention here that, as was first established by Dirac [26], the physical state $|\Phi\rangle$ can be written as

$$\begin{aligned}
|\Phi\rangle &\equiv \left|\overline{\Psi}(\mathbf{y}) \Psi(\mathbf{y}\prime)\right\rangle \\
&= \overline{\psi}(\mathbf{y}) \exp\left(iq \int_{\mathbf{y}\prime}^{\mathbf{y}} dz^i A_i(z)\right) \psi(\mathbf{y}\prime) |0\rangle, \quad (11)
\end{aligned}$$

where the line integral is along a spacelike path on a fixed time slice, and $|0\rangle$ is the physical vacuum state. Notice that the charged matter field together with the electromagnetic cloud (dressing) which surrounds it, is given by $\Psi(\mathbf{y}) = \exp\left(-iq \int_{C_{\xi \mathbf{y}}} dz^\mu A_\mu(z)\right) \psi(\mathbf{y})$. Thanks to our path choice, this physical fermion then becomes $\Psi(\mathbf{y}) = \exp\left(-iq \int_0^{\mathbf{y}} dz^i A_i(z)\right) \psi(\mathbf{y})$. In other terms, each of the states ($|\Phi\rangle$) represents a fermion-antifermion pair surrounded by a cloud of gauge fields to maintain gauge invariance.

Taking into account the above Hamiltonian structure, we observe that

$$\begin{aligned}
\Pi_i(x) \left|\overline{\Psi}(\mathbf{y}) \Psi(\mathbf{y}')\right\rangle &= \overline{\Psi}(\mathbf{y}) \Psi(\mathbf{y}') \Pi_i(x) |0\rangle \\
&\quad + q \int_{\mathbf{y}}^{\mathbf{y}'} dz_i \delta^{(3)}(\mathbf{z} - \mathbf{x}) |\Phi\rangle. \\
&\qquad\qquad\qquad\qquad\qquad (12)
\end{aligned}$$

Having made this observation and since the fermions are taken to be infinitely massive (static) we can substitute Δ by $-\nabla^2$ in Eq. (8). Therefore, the expectation value $\langle H\rangle_\Phi$ is expressed as

$$\langle H\rangle_\Phi = \langle H\rangle_0 + \langle H\rangle_\Phi^{(1)} + \langle H\rangle_\Phi^{(2)}, \qquad (13)$$

where $\langle H\rangle_0 = \langle 0| H |0\rangle$. The $\langle H\rangle_\Phi^{(1)}$ and $\langle H\rangle_\Phi^{(2)}$ terms are given by

$$\langle H\rangle_\Phi^{(1)} = -\frac{1}{2}\frac{b}{\xi^2} \langle\Phi| \int d^3x \Pi_i \frac{\nabla^2}{(\nabla^2 - M^2)} \Pi^i |\Phi\rangle, \quad (14)$$

and

$$\langle H\rangle_\Phi^{(2)} = \frac{M^2}{2} \langle\Phi| \int d^3x \Pi_i \frac{1}{(\nabla^2 - M^2)} \Pi^i |\Phi\rangle, \qquad (15)$$

where $M^2 \equiv \frac{m^2}{\xi^2} = \frac{m^2}{b + g^2 \mathcal{B}^2}$. Using Eq. (12), the $\langle H\rangle_\Phi^{(1)}$ and $\langle H\rangle_\Phi^{(2)}$ terms can be rewritten as

$$\begin{aligned}
\langle H\rangle_\Phi^{(1)} &= -\frac{1}{2}\frac{bq^2}{(b + g^2\mathcal{B}^2)} \int d^3x \int_{\mathbf{y}}^{\mathbf{y}'} dz_i' \delta^{(3)}(\mathbf{x} - \mathbf{z}') \\
&\quad \times \left(1 - \frac{\nabla^2}{M^2}\right)_x^{-1} \int_{\mathbf{y}}^{\mathbf{y}'} dz^i \delta^{(3)}(\mathbf{x} - \mathbf{z}), \qquad (16)
\end{aligned}$$

and

$$\begin{aligned}
\langle H\rangle_\Phi^{(2)} &= \frac{M^2 q^2}{2} \int d^3x \int_{\mathbf{y}}^{\mathbf{y}'} dz_i' \delta^{(3)}(\mathbf{x} - \mathbf{z}') \\
&\quad \times \left(\nabla^2 - M^2\right)_x^{-1} \int_{\mathbf{y}}^{\mathbf{y}'} dz^i \delta^{(3)}(\mathbf{x} - \mathbf{z}). \quad (17)
\end{aligned}$$

Following our earlier procedure [27, 28], we see that the potential for two opposite charges located at $\mathbf{y}$ and $\mathbf{y}'$ takes the form

$$\begin{aligned}
V &= -\frac{q^2 b}{4\pi(b + g^2\mathcal{B}^2)} \frac{e^{-\left(\sqrt{\frac{m^2}{b + g^2\mathcal{B}^2}}\right)L}}{L} \\
&\quad + \frac{q^2 m^2}{8\pi(b + g^2\mathcal{B}^2)} \ln\left(1 + \frac{\Lambda^2}{m^2}(b + g^2\mathcal{B}^2)\right) L, (18)
\end{aligned}$$

where Λ is a cutoff and $|\mathbf{y} - \mathbf{y}'| \equiv L$. Hence we see that the static potential profile displays a confining behavior. Notice that expression (18) is spherically symmetric, although the external fields break the isotropy of the problem in a manifest way. The Yukawa-type component to the potential above vanishes whenever $b = 0$. It actually signals the contribution of a scalar boson (the spin-0 mode of S_μ) to the interparticle potential. And, from our Lagrangian (1), this scalar boson shows up if and only if $b \neq 0$. So, this Yukawa screening appears as a byproduct of a dynamical spin-0 torsion. As previously pointed out, for $m^2 \ll m_0^2$, which is the physically acceptable regime, this screening fades off.

It is now important to give a meaning to the cutoff Λ. To do that, we should recall that our effective model for the electromagnetic field is an effective description that comes out upon integration over the torsion, whose excitation is massive. $1/m$, the Compton wavelength of this excitation, naturally defines a correlation distance. Physics at distances of the order or lower than $1/m$ must necessarily take into account a microscopic description

of torsion. This means that, if we work with energies of the order or higher than m, our effective description with the integrated effects of S^μ is no longer sensible. So, it is legitimate that, for the sake of our analysis, we identify Λ with m. Then, with this identification, the potential of Eq. (18) takes the form below:

$$
\begin{aligned}
V = & -\frac{q^2 b}{4\pi \left(b + g^2 \mathcal{B}^2\right)} \frac{e^{-\left(\sqrt{\frac{m^2}{b + g^2 \mathcal{B}^2}}\right) L}}{L} \\
& + \frac{q^2 m^2}{8\pi \left(b + g^2 \mathcal{B}^2\right)} \ln \left(1 + b + g^2 \mathcal{B}^2\right) L. \quad (19)
\end{aligned}
$$

An immediate consequence of this is that for $b = 0$ the screening term (encoded in the Yukawa potential) disappears of the static potential profile, describing an exactly confining phase [22].

B. Electric case

We now extend what we have done to the case $v^{0i} = 0$ and $v^{ij} \neq 0$ (referred to as the electric one in what follows). Thus, the corresponding Lagrangian is given by

$$
\mathcal{L}_{eff} = -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} + \frac{g^2}{8} v^{ij} f_{ij} \frac{\Delta}{\left(b\Delta + m^2\right)} v^{kl} f_{kl}, \quad (20)
$$

with $\mu, \nu = 0, 1, 2, 3$ and $i, j, k, l = 1, 2, 3$. Following the same steps employed for obtaining (18), we now carry out a Hamiltonian analysis of this model. First, the canonical momenta following from Eq.(20) are $\Pi^\mu = f^{\mu 0}$, which results in the usual primary constraint $\Pi^0 = 0$ and $\Pi^i = f^{i0}$. Defining the electric and magnetic fields by $E^i = f^{i0}$ and $B^i = -\frac{1}{2} \varepsilon^{ijk} f_{jk}$, respectively, the canonical Hamiltonian can be written as

$$
\begin{aligned}
H_C = & \int d^3 x \left\{ -A_0 \partial_i \Pi^i + \frac{1}{2} \mathbf{\Pi}^2 + \frac{1}{2} \mathbf{B}^2 \right\} \\
& - \frac{g^2}{8} \int d^3 x \left\{ \varepsilon_{ijm} \varepsilon_{k\ln} v^{ij} B^m \frac{\Delta}{\left(b\Delta + m^2\right)} v^{kl} B^n \right\}.
\end{aligned}
$$
$$(21)$$

Time conservation of the primary constraint leads to the secondary constraint, $\Gamma_1(x) \equiv \partial_i \Pi^i = 0$, and the time stability of the secondary constraint does not induce more constraints, which are first class. It should be noted that the constrained structure for the gauge field is identical to the usual Maxwell theory. Therefore, the corresponding expectation value is given by

$$
\langle H \rangle_\Phi = \frac{1}{2} \langle \Phi | \int d^3 x \Pi^2 | \Phi \rangle. \quad (22)
$$

As was explained in [29], expression (22) becomes

$$
V = -\frac{q^2}{4\pi} \frac{1}{|\mathbf{y} - \mathbf{y}'|}, \quad (23)
$$

which it is just the Coulomb potential.

Our understanding on the remarkable distinction between the cases of an external electric and magnetic fields is as follows. Back to the Lagrangian (1), the torsion-electromagnetic field interaction term can be also written as

$$
\mathcal{L}_{int} = -g \left(\partial_\lambda S^\lambda\right) \mathbf{E} \cdot \mathbf{B}, \quad (24)
$$

up to a surface term. By splitting the classical background and the quantum fluctuation of the electromagnetic field,

$$
\mathbf{E} = \langle \mathbf{E} \rangle + \mathbf{e}, \quad (25)
$$

and

$$
\mathbf{B} = \langle \mathbf{B} \rangle + \mathbf{b}, \quad (26)
$$

we readily see that:

(i) In the case the background is purely magnetic, $\langle \mathbf{E} \rangle = 0$, torsion couples to the fluctuation $\mathbf{e}$ via $\langle \mathbf{B} \rangle$. Since we are seeking the interparticle potential in the static regime the interaction term with $\langle \mathbf{B} \rangle$ present is $-g \left(\partial_\lambda S^\lambda\right) \langle \mathbf{B} \rangle \cdot \mathbf{e} = -g \left(\partial_\lambda S^\lambda\right) \langle \mathbf{B} \rangle \cdot \nabla \varphi$, so that, we are lead to conclude that it is the interchange of the scalar φ the responsible for the Yukawa-like piece of the potential.

(ii) For an electric background, $\langle \mathbf{B} \rangle = 0$, the interaction term with $\langle \mathbf{E} \rangle$ present reads $-g \left(\partial_\lambda S^\lambda\right) \langle \mathbf{E} \rangle \cdot \mathbf{b} = -g \left(\partial_\lambda S^\lambda\right) \langle \mathbf{E} \rangle \cdot \left(\nabla \times \mathbf{A}\right)$. In this case, the contribution to the potential is due to an $\mathbf{A}$-exchange and this is why no Yukawa-term shows up.

III. FINAL REMARKS

In summary, by using the gauge-invariant but path-dependent formalism, we have extended our previous analysis about the static potential for a theory which includes both spin-1 and spin-0 states for the axial torsion field S_μ coupled to photons, in the case when there are nontrivial constant expectation values for the gauge field strength $F_{\mu\nu}$.

It was shown that in the case when $\langle F_{\mu\nu} \rangle$ is electric-like no unexpected features are found. Indeed, the resulting static potential remains Coulombic. More interestingly, it was shown that when $\langle F_{\mu\nu} \rangle$ is magnetic-like the potential between static charges displays a Yukawa piece plus a linear confining piece. We stress here the role played by the spin-0 state of the torsion field S_μ in yielding the Yukawa potential. An analogous static potential profile in axionic electrodynamics may be recalled [23].

IV. ACKNOWLEDGMENTS

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